

Epistemology: A Case Study

UE RECHERCHE | IMT ATLANTIQUE

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Slides partly adapted from Vincent Gripon's slides (2022)

Epistemology : reflection on what knowledge is, and on what allows us to consider it justified, particularly in the sciences.

It raises questions such as :

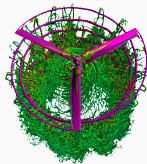
- **Opinion vs. knowledge?** Being convinced is not enough to be right.
- **What counts as proof?** Observation, experiment, and mathematical demonstration do not establish things the same way.
- **How far can we trust a result?** Under what assumptions, with what limits?
- **How is knowledge corrected?** What role do methods and the scientific community play?

Research produces new knowledge; epistemology examines how it is built and justified.

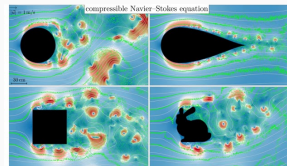
Goal of this session : make this less abstract, using one concrete case.

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}, \quad \nabla \cdot \mathbf{u} = 0$$

It describes how a fluid's velocity $\mathbf{u}$ and pressure p evolve over time.



Rotor / wind-turbine wake vortices (NASA, N. Chaderjian)



Flow around different shapes (CFD simulation)

The Millennium Prize question is **not** about using the equation: engineers already do that routinely. It asks whether every mathematically admissible solution stays smooth forever, or whether some can blow up.

On 8 September 2026, OpenAI announced a mathematical proof concerning the three-dimensional Navier–Stokes equations.

What is documented

- About 10,000 AI agents (OpenAI's own figures)
- A paper of about 166 pages
- A formalization in the proof assistant **Lean**, made public
- OpenAI states it is not claiming the prize

What is disputed or still open

- A priority dispute with other researchers
- The exact scope of the statement addressed
- Independent verification still ongoing
- Clay's rules require peer-reviewed publication **and** 2 years of scrutiny

You have a documentary pack about this announcement (articles, excerpts, reactions).

Moodle : Parcours recherche → UE Recherche → Ressources → "Dossier sur Navier-Stokes - OpenAI".

Instructions

How do these documents relate to epistemology?

Write your comment on the Framapad (link on Moodle).

What is your conclusion ?

Reading the documents as a set brings out named ideas from epistemology.

- **Formal, deductive certainty:** Lean checks, purely mechanically, that each step follows from the axioms. That gives certainty within a formal system, not certainty about the world. (*Lean documentation, GitHub repository.*)
- **Epistemic opacity:** a machine-checked proof establishes that a statement is true, not that anyone understands why. Several mathematicians report the paper is extremely hard to read, even ones who believe it is correct. (*NPR.*)

- **Genealogy of knowledge:** the strategy behind the result did not appear from nothing. Both Buckmaster and Fefferman's own community trace it to a multi-year program by Córdoba and Martínez-Zoroa. (*Buckmaster's statement, Quanta Magazine.*)
- **Provenance and credit:** whether unpublished work reached OpenAI before its announcement is disputed and, from the documents alone, unresolved. (*Buckmaster's statement.*)
- **What a proof is for:** obtaining a result, understanding why it is true, and being able to reuse its ideas are three different things. (*Grant Sanderson, guest post on Terence Tao's blog.*)

Summary : source and epistemological takeaway

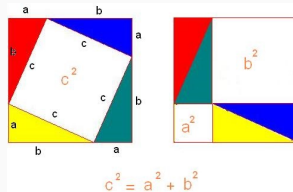
Source	Epistemological takeaway
Lean documentation, GitHub repository	Formal validity within a system, not truth about the world
NPR	Opacity gap : correctness is not comprehension
Buckmaster's statement, Quanta Magazine	Ideas have a genealogy ; credit and provenance matter
Grant Sanderson (guest post, Tao's blog)	Mathematical value beyond proof production

(Scientific) knowledge, or suspicion ?

Proposition

Under the ZFC axioms, the Pythagorean theorem.

Refutable statement...

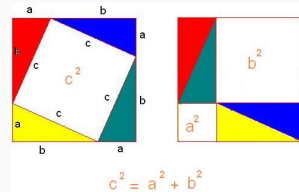


What counts as scientific knowledge ?

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Under the ZFC axioms, the Pythagorean theorem.

Refutable statement, proved by deduction...



What counts as scientific knowledge ?

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The Earth is "round".

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What counts as scientific knowledge ?

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The Earth is "round".

Refutable statement, proved by deduction, by observation...



Proposition

The Earth is "flat".

Refutable statement, proved by deduction, by observation...



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The Earth is "flat".

Refutable statement, coherent with other knowledge,
proved by deduction, by observation...



Proposition

The diversity of life is explained by lineages diversifying and being selected according to their environment.

Refutable statement, coherent with other knowledge,
proved by deduction, by observation...



Proposition

The diversity of life is explained by lineages diversifying and being selected according to their environment.

Refutable statement, coherent with other knowledge, proved by deduction, by observation, by predictive power...



Proposition

Some people have the gift of divination.

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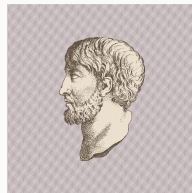
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Any body immersed in a fluid at rest experiences an upward force equal to the weight of the fluid displaced.

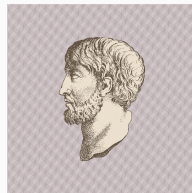
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Hydroxychloroquine combined with antibiotics is effective at reducing mortality from COVID-19.

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Hydroxychloroquine and azithromycin as a treatment of COVID-19: results of an open-label non-randomized clinical trial

Philippe Gautret^{a,b}, Jean-Christophe Lagier^{a,c}, Philippe Parola^{a,b}, Van Thuan Hoang^{a,b,d}, Line Meddeb^a, Morgane Mailhe^a, Barbara Doudier^a, Johan Courjon^{c,d}, Valérie Giordanengo^b, Vera Esteves Vieira^a, Hervé Tissot Dupont^{a,c}, Stéphane Honoré^d, Philippe Colson^{a,c}, Eric Chabrière^{a,c}, Bernard La Scola^{a,c}, Jean-Marc Rolain^{a,c}, Philippe Brouqui^{a,c}, Didier Raoult^{a,c}.

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Refutable statement, coherent with other knowledge, proved by deduction, by observation, by demonstrated predictive power, by experimentation or *rigorous* clinical trials...

Why is it so difficult to construct knowledge ?

- **Believing** : holding something as true, regardless of whether it actually is.
- You can believe something true *without knowing it* : you also need a justification, not just a lucky coincidence.

💡 The stuck thermometer

A broken thermometer, stuck at 20°C, happens to show the correct temperature on a day when it really is 20°C. The belief that it is 20°C is **true**, but not **justified** by a working instrument. You are right, without really knowing.

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Gettier (1963) : he showed that a belief can be both **true** and **justified** without actually constituting genuine knowledge.

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A broader inventory of cognitive biases (Wikipedia) is given on the next slide.

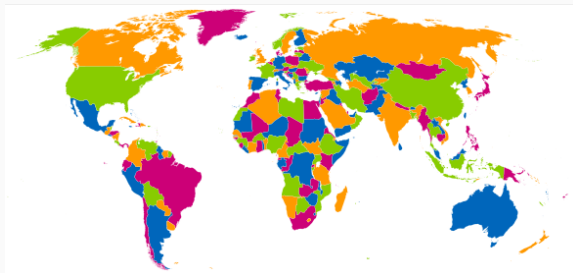
Liste non exhaustive de biais cognitifs bien connus (Wikipedia)

Biais d'attention	Effet de récence	Simple exposition
Effet de primauté	Oubli fréquence de base	Appel à la probabilité
Aversion à la dépossession	Biais d'ancrage	Biais d'attribution
Biais d'autocomplaisance	Biais d'engagement	Biais d'équiprobabilité
Biais d'immunité à l'erreur	Biais de confirmation	Biais de normalité
Biais de présentéisme	Biais de proportionnalité	Biais de statu quo
Biais égocentrique	Biais rétrospectif	Croyance en un monde juste
Effet d'ambiguïté	Effet de halo	Effet Dunning-Kruger
Effet Ikea	Effet Stroop	Effet râteau
Erreur fondamentale d'attribution	Illusion de savoir	Illusion monétaire
Illusion de transparence	Loi de l'instrument	Sophisme génétique
Supériorité illusoire	Tache aveugle à l'égard des préjugés	Biais de confirmation d'hypothèse
Biais d'évaluation de probabilités	Biais de représentativité	Biais de disponibilité
Biais d'appariement	Biais du survivant	Dissonance cognitive
Effet rebond	Illusion des séries	Perception sélective
Réification du savoir	Effet leurre	Coût irrécupérable
Biais d'optimisme	Effet Barnum	...

Problem : how do I know it is science, if I cannot check it myself ?

💡 The four colour theorem

Every planar graph can be coloured with at most 4 colours. Proved (1976) with computer assistance to check a huge number of cases : few people can verify every step by hand, yet the mathematical community accepts the result.



What matters is not “can I check it myself”, but whether an **independent verification process** exists, accessible to people other than the author.

- **Producing** a proof (here, by AI agents) : a heuristic research phase, with no a priori guarantee of correctness.
- **Checking** a proof with an assistant like **Lean** : the software mechanically checks that every step is rigorously derived from the previous ones, from the axioms and the **formalized statement**.

What Lean guarantees

No logical error in the deduction chain, as it was formalized.

What this alone does not guarantee

That the formalized statement matches the intended mathematical problem. This still needs to be checked by competent humans.

Checking is not understanding : a text can be formally validated before the community has identified the mathematical idea it contains.

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8. (quasi-)Scientific consensus

The longer a statement is the subject of consensus, while also generating real interest from scientific communities, the more it constitutes a **robust form of scientific knowledge**.

Which apparent guarantees are not enough ?

Some signals give an impression of scientific seriousness without being a guarantee of it :

- ❑ The existence of an article or preprint online, without independent review.
- ❑ The reputation or authority of the author or institution.
- ❑ Broad, repeated media coverage.
- ❑ A tool that looks rigorous (a long document, a software formalization), taken in isolation.

What makes a practice scientific is not a label, but how a claim is **put to the test** : can it be tested, replicated, discussed, and revised when it turns out to be wrong ?

1. Intelligent design
2. Astrology
3. Sophrology
4. Ufology
5. Homeopathy
6. Naturopathy
7. "Occult sciences"
8. Psychoanalysis
9. Radiesthesia (dowsing)
10. Water memory
11. Animal magnetism
12. ...

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Not guarantees of scientific knowledge :

1. University-level programs (even accredited diplomas)
2. Peer-reviewed literature and journals
3. Scientific endorsements
4. Popular common sense or majority opinion

What benchmarks help examine a claim and revise one's judgment ?

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None of these benchmarks excuses examining the particular case. Error correction is **not automatic** : it depends on practices actually carried out. A **justified doubt** rests on precise reasons ; a doubt that rejects everything on principle is no more rigorous than blind trust.

In the public sector

Academic (universities, engineering schools...) :

- Senior lecturers, professors
- Research engineers
- Postdocs
- PhD students
- Students (you!)

Research institutes (CNRS, Inria, Inserm...) :

- Research fellows, research directors

In the private sector

R&D : engineers, researchers

Private institutes (e.g. Institut Pasteur)



UE Recherche puts you inside this very ecosystem, at a small scale : one exploratory research project, run like preparing a conference paper : choosing a topic, writing an *extended abstract*, **peer review** by other students, then a public presentation.

Everything we discussed today, refutability, the gap between checking and understanding, what a review actually guarantees, revising a claim when it doesn't hold up, is not just about Navier–Stokes : it is exactly what you will practice firsthand, on **your own** claims, read critically by your peers.

In short, as things stand : a published, formalized claim, with an ongoing priority dispute and independent verification still in progress.

In your Research Course project, you will also be evaluating claims you cannot always fully check yourselves : a paper, an experimental result, technical documentation, an answer from generative AI...

Some transferable habits : separate the claim from its proof ; look for **independent** verifications ; distinguish **checking** from **understanding**.

Not every situation maps onto this example, but the questions worth asking do carry over.

- Edmund L. Gettier, "Is Justified True Belief Knowledge?", *Analysis*, 23(6), 1963.
- OpenAI, "On the Navier–Stokes Millennium Prize Problem", <https://openai.com/index/navier-stokes-solution/>, 8 September 2026.
- K. Kakaes, "AI Has Solved One of Math's \$1 Million Millennium Prize Problems", *Quanta Magazine*, 8 September 2026.
- "OpenAI Claims Its AI Proved a Navier-Stokes Blowup – Mathematicians Question Priority and Verification", *XenoSpectrum*, September 2026.
- Wikipedia, *Navier–Stokes existence and smoothness* and *Navier–Stokes priority controversy*, accessed 19 September 2026.

This beamer was partly generated using an LLM.