A NOVEL HIGH-ORDER NUMERICAL SCHEME FOR FLUID DYNAMICS

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Abstract

We propse a high-order compact finite diference scheme for the 2D incompressible Navier-Stokes equations with periodic boundary conditions. The method combines forth-order spatial discretization with second-order implicite time integration and a simplifed treatment of the nonlinear convective term. Numerical tests show that the proposed scheme consistenly achieves lower L_2 velocity errors than the standard FFD method, demonstating superior accurcy and robustness. These results indicates that our method outperforms existing numerical aproaches.

INTRODUCTION

Accurate and efficient numerical solutions of the incompressible Navier-Stokes equations are essential for a wide range of engineering and scientific applications, including aerodinamics simulations, enviromental flows, and industrial process modellings [3]. Traditional finite difference methods, such as the Fast Finite Difference (FFD) scheme, are widely used due to their simplicity and computational efficency.
High-order compact schemes has been proposed to improve spatial accuracy while retaining a compact stensil [1, 2]. Some other approachs, like turbulence modeling in complex geometries, has also been investigated in related contexts [4]. Anyway, we propose a fourth-order compact finite difference scheme in space, combined with a second-order implicite time integration, to solve the 2D incompressible Navier-Stokes equations under periodic boundary conditions. We first describe the numerical method, including the discretization and implementation details, then present the numerical results with error analyse for selected grid resolutions, and finally discuss the performance of the proposed scheme in comparisson to standard methods and highlights the main conclusions of the study.

NUMERICAL METHOD

The governing 2D incompressible Navier-Stokes equations are

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u}, \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0, \tag{2}$$

where $\mathbf{u}$ is the velocity field, p the pressure, and ν the kinematic viscosity.
We implement a fourth-order compact finite difference scheme in space, combined with a second-order implicit time integration. Periodic boundary conditions are applied in both directions.
Discretizing Eq. (1) in space and time, the system can be written as

$$\mathbf{U}^{n+1} - \mathbf{U}^n + \Delta t \left[(\mathbf{U}^n \cdot \nabla)\mathbf{U}^{n+1} + \nabla P^{n+1} - \nu \nabla^2 \mathbf{U}^{n+1} \right] = 0, \tag{3}$$

where $\mathbf{U}^n$ and P^n denote the discrete velocity and pressure vectors at time step n.
Then we can show that

$$(\mathbf{U}^n \cdot \nabla)\mathbf{U}^{n+1} \approx \frac{1}{2} \nabla \left(|\mathbf{U}^n|^2 \right), \tag{4}$$

which allows us to simplify the discrete momentum equation and obtain the resulting scheme

$$\mathbf{U}^{n+1} - \mathbf{U}^n + \frac{\Delta t}{2} \nabla \left(|\mathbf{U}^n|^2 \right) + \Delta t \nabla P^{n+1} - \Delta t \nu \nabla^2 \mathbf{U}^{n+1} = 0. \tag{5}$$

RESULTS

To evaluate the performance of the proposed numerical scheme, we computed the L_2 errors of velocity for a range of grid resolutions. Figure 1 compares the convergence behavior of our method against the standard FFD method [**?**].
The results clearly indicate that our method yields lower L_2 errors for all tested grid resolutions, confirming its effectiveness and robustness.

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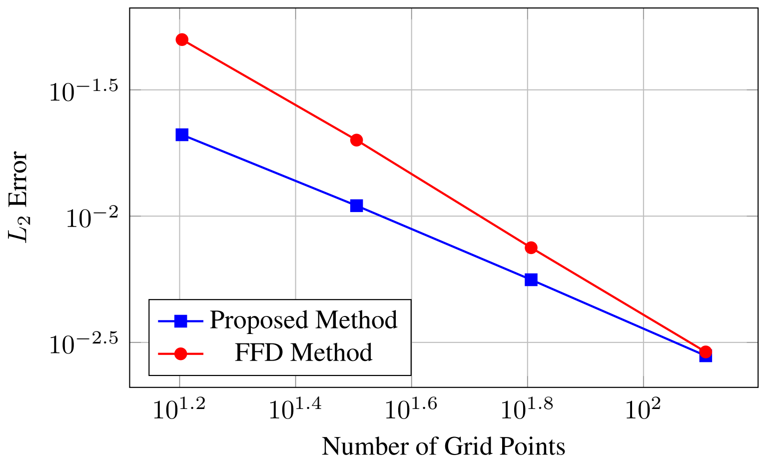

Figure 1: Comparison of L_2 velocity errors between the proposed numerical scheme and the standard FFD method for different grid resolutions.

DISCUSSION

Based on the L_2 errors reported in Figure 1, we claim that the proposed numerical scheme outperforms the FFD method across all tested grid resolutions. Our method consistently shows lower errors, demonstrating its superior accuracy and robustness for typical discretizations.

References

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